Problems of Derivability of functions

1) Find out the continuity and derivability of the following function at the point $x = 1$:

$$ f(x) = \begin{cases} 
15x + 6 & \text{if } x < 1 \\
3x^2 + 9x + 10 & \text{if } x \geq 1 
\end{cases} $$

2) Find out the continuity and derivability of the following function on $\mathbb{R}$:

$$ f(x) = 3\sqrt[3]{x + 1} $$

3) Find out the continuity and derivability of the following function at the point $x = -3$:

$$ f(x) = \begin{cases} 
2x^3 - 82 & \text{if } x \leq -3 \\
4x^2 + 73x + 47 & \text{if } x > -3 
\end{cases} $$

4) Find out the continuity and derivability of the following function on $\mathbb{R}$:

$$ f(x) = |x - 7| + |x + 3| $$

5) Find out the continuity and derivability of the following function at the point $x = 3$:

$$ f(x) = \begin{cases} 
2x^2 - 15x + 30 & \text{if } x < 3 \\
\frac{3}{x - 2} & \text{if } x \geq 3 
\end{cases} $$

6) Find out the continuity and derivability of the following function on $\mathbb{R}$:

$$ f(x) = |x - 4| - 4x $$

7) Find out the value of the parameter $p$ for which the following function is continuous and has derivative on $\mathbb{R}$.

$$ f(x) = \begin{cases} 
18 - px^2 & \text{if } x \leq 1 \\
\frac{72}{px} & \text{if } x > 1 
\end{cases} $$

8) Find out the value of the parameters $m$ and $p$ for which the following function is continuous and has derivative on $\mathbb{R}$.

$$ f(x) = \begin{cases} 
x^2 - 5x + m & \text{if } x \leq -1 \\
-x^2 + px & \text{if } x > -1 
\end{cases} $$
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9) Find out the value of the parameters \( p \) and \( r \) for which the following function is continuous and has derivative on \( \mathbb{R} \).

\[
f(x) = \begin{cases} 
5x^3 + 3x & \text{if } x < 2 \\
px + r & \text{if } x \geq 2 
\end{cases}
\]

10) Find out the value of the parameters \( h \) and \( k \) for which the following function is continuous and has derivative on \( \mathbb{R} \).

\[
f(x) = \begin{cases} 
8x + he^{-4} & \text{if } x < 4 \\
3x^2 + kx & \text{if } x \geq 4 
\end{cases}
\]

11) Find out the value of the parameters \( h \) and \( k \) for which the following function is continuous and has derivative on \( \mathbb{R} \).

\[
f(x) = \begin{cases} 
-3x^2 + hx + k & \text{if } x < 1 \\
x + 9 & \text{if } x \geq 1 
\end{cases}
\]
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Answers:
1) Has a jump discontinuity and hasn't derivative.
2) Continuous on $\mathbb{R}$. Has derivative on $\mathbb{R} - \{-1\}$ (vertical tangent).
3) Continuous and hasn't derivative.
4) Continuous on $\mathbb{R}$. Has derivative on $\mathbb{R} - \{-3, 7\}$ (peak points).
5) Continuous and has derivative.
6) Continuous on $\mathbb{R}$. Has derivative on $\mathbb{R} - \{4\}$ (peak point).
7) $p = 6$
8) $m = 2, \ p = -9$
9) $p = 63, \ r = -80$
10) $h = 16, \ k = 0$
11) $h = 7, \ k = 6$